

# TotalRisk

## Statistics for modelling the risk of financial institutions

**Innovation area:** Finance

**Key Innovator:** Kjersti Aas

**Partners:** DNB, NR, UiO, NTNU

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### Scope:

Development of the pair copula construction as a preferred method for the analysis of multiple variables with strong tail dependence in financial applications

### Results in 2012 and plans:

Risk management confronts us with heavy-tailed distributions, rapid changes and complex dependencies. This forces us to look beyond standard statistical models to develop more sophisticated methodology. We need to consider both supervised approaches, when much is known in detail about the variables at play, and un-supervised contexts, where adaptive modelling strategies are required to automatically capture variation from data. Our objective is to renew the tools used for total risk modelling in financial institutions, producing more reliable and useful estimates of the risk. Complex dependency and interactions are key aspects. Though extreme events for financial data manifest themselves through high gains and/or losses, very often it is the *co-movement* of underlying instruments in times of crises that triggers such events; the so-called perfect storm scenario. Hence, the modeling of the joint occurrence of extremes ought to be of prime concern, when modeling risk types as credit risk, market and operational risk. Dependence structures are often non-linear, requiring new statistical methods, such as copula-based approaches. We develop new ways to construct multivariate distributions from smaller components, building a theory beyond conditional independence. The distributions of financial returns, credit losses, and operational losses all have multivariate heavy tails. Some of them are skewed, with one tail heavier than the other. We develop models for heavy-tailed and skewed phenomena. Our research results will be validated within our partners, tested and confronted, with the aim to become part of best practice in the years to come.

In 2012 we have worked in five main areas:

(i) Pair-copula constructions: The focus of this activity is to improve the state-of-the art methods for modelling the dependency in the tails of a multivariate distribution.

(ii) Copula model estimation and evaluation: In this sub-project we study different aspects connected to estimation and model evaluation of copulas.

(iii) Rehabilitation of illegal correlation matrices: The aim of this activity is transform a non-positive definite correlation matrix into a legal one using Bayesian methods.

(iv) Improving simulation accuracy: In many financial risk management applications, Monte Carlo simulations have become the standard approach for computing the desired risk measures. However, since one usually is interested in quantiles far out in the tail of the distribution, naïve Monte Carlo simulation would require a very large number of simulations. For DNB's current portfolio not even 5 million simulations is sufficient to obtain the desired accuracy of 99.97% VaR.

(v) For rare event estimation problems, importance sampling (IS) often provides an efficient means of generating low variance estimates. In 2009, (sfi)<sup>2</sup> developed a new IS technique for VaR estimation and Expected Shortfall-based capital allocation that was shown to work well for all sorts of real-world credit portfolios (Reitan & Aas, 2010). In 2012 we have adapted this methodology to DNB's current credit risk model. The result has been implemented in DNBs risk management systems.

In 2013 subprojects (i) and (ii) will continue. These are described in more details below. If the external evaluation of DNBs total risk model taking place in October and November 2012 identifies scientific weaknesses of this model, we might also start one or two new subprojects. However, these will in case not be defined before early 2013.

Pair-copula constructions: In 2013 and 2014 the aim of this activity is twofold:

(i) We want to study the relationship between Bayesian Belief Networks and pair-copula constructions.

(ii) We want to develop new methods for truncating pair-copula constructions.

We hope to produce one paper on each of these themes. In what follows we describe the problems in a bit more detail:

Pair-copula constructions bear many similarities with Bayesian Belief Networks (BBNs). The latter are very much used in practical applications. However, they have almost only been restricted to the multivariate normal or discrete distributions. When faced with continuous data that cannot be captured well with the multivariate Gaussian, the vast majority of work first discretize the data, and then take advantage of the methods that has been made for the discrete case. Copulas offer a flexible mechanism for modelling continuous distributions. Hence, the two frameworks thus complement each other in a way that offers opportunities for fruitful synergic innovations.

For pair-copula constructions (PCCs) to be really useful in practice, one needs to be able to fit such structures to data with more than 20 dimensions. However, a problem with the PCC is that the computational effort required to estimate all parameters grow exponentially with the dimension. Hence, we have previously studied methods for truncating a PCC, where we by a K-truncated PCC means a pair-copula construction for which all pair-copulas with a conditioning set equal to or larger than K are replaced by independence copulas (Brechmann et.

al., 2012). The previously developed truncation methods are not optimal. Hence, for in 2013 we will continue this activity and develop alternative criteria for how to truncate a PCC.

Model evaluation: We have previously shown (Grønneberg and Hjort, 2012) that the arguments leading to the classical AIC does not hold for the case of parametric copula models using the maximum pseudo likelihood procedure. Moreover, we have derived a proper correction, denoted the CIC, the Copula Information Criterion which is fundamentally different from the AIC erroneously used by practitioners. In 2013, we will investigate whether another model validation tool, the focused information criterion (FIC), may be successfully applied to pair-copula constructions. The FIC (Claeskens and Hjort, 2003) is a method for selecting the most appropriate model among a set of competitors for a given data set. Unlike most other model selection strategies, like the AIC, the FIC does not attempt to assess the overall fit of candidate models, but focuses attention directly on the parameter of primary interest with the statistical analysis. This parameter might e.g. be the Value-at-Risk.

Innovation and technology transfer: In addition to the activities described above, our aim for 2013 is trying to identify the added value of some of the tools previously developed in the TotalRisk project to innovation in Norwegian and international financial institutions. More specifically, we will concentrate on two tools:

- (i) The Bayesian tool for rehabilitating an illegal correlation matrix.
- (ii) The pair-copula constructions

As far as the first innovation is concerned, and earlier version of this tool is already operational at DNB. Regarding the pair-copula constructions, we know from the literature that these structures have been used for e.g. risk management and portfolio optimization. However, we want to more systematically investigate how the PCCs are used in Norwegian and international financial institutions.

Pair-copula workshop: In December 2009 we hosted an international workshop on pair-copula constructions. This workshop was very successful. Since then, the "PCC community" has significantly increased (the current number of citations of the paper Aas et. al, 2009 at Google Scholar is currently 233). Hence, we believe that a new workshop on pair-copula constructions will gather many participants. The plan is to organize such a workshop either late 2013 or spring 2014.

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